Quiz 10 solution

$$x\frac{dy}{dx} + 2y = \frac{\sin(2x)}{x} + x$$

$$\frac{dy}{dx} + \frac{2}{x}y = \frac{\sin(2x)}{x^2} + 1$$

so $p(x) = +\frac{2}{x}$, $q(x) = \frac{\sin(2x)}{x^2} + 1$

$$I(x) = e^{\int \frac{2}{x}dx}$$

$$= e^{2\ln x}$$

$$= e^{\ln(x^2)}$$

$$= x^2$$

so DE becomes:

$$\frac{d}{dx}(yx^2) = \sin(2x) + x^2$$

$$\int \frac{d}{dx}(yx^2)dx = \int \sin(2x) + x^2\,dx$$

$$yx^2 = -\frac{\cos(2x)}{2} + \frac{x^3}{3} + c$$

when $x = \frac{\pi}{4}$, $y = 0$, so

$$0 = -\frac{\cos(\pi/2)}{2} + \frac{\pi^3}{3\times 64} + c \Rightarrow c = -\frac{\pi^3}{192}$$

soln is $\boxed{yx^2 = -\frac{\cos(2x)}{2} + x^3/3 - \pi^3/192}$