Math 20 — **Quiz 2**

Name: — **Date: 1/29/2014**

Directions: Calculators are allowed. Show all your working! Use the back of the page if you run out of space.

Consider the definite integral

$$\int_1^5 \frac{6}{2x+1}dx.$$

1. Find approximations to the area underneath this curve using $n = 4$ equal subintervals and:

(a) rectangles with their left-hand edges touching the curve (i.e., an "upper" sum);

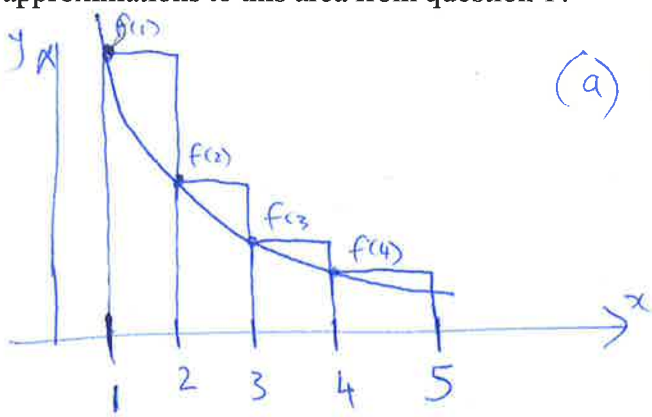
(b) the trapezoidal rule;

(c) Simpson's rule.

(Note: you can make me smile by drawing sketches of the areas calculated by each method above.)

2. The true area underneath the curve is approximately 3.89785. How could you improve each of your approximations to this area from question 1?

①



(a) $A_U = 1\times f(1) + 1\times f(2) + 1\times f(3) + 1\times f(4)$

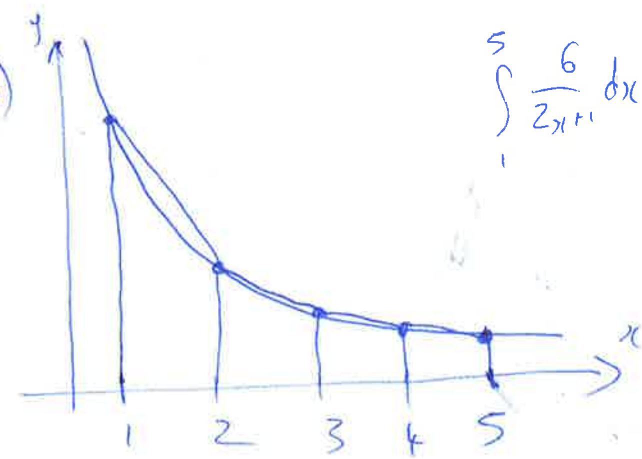
$= \frac{6}{2+1} + \frac{6}{4+1} + \frac{6}{6+1} + \frac{6}{8+1}$

$= \frac{6}{3} + \frac{6}{5} + \frac{6}{7} + \frac{6}{9}$

$= 6\left[\frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9}\right]$

$= 4.7238$

(b)



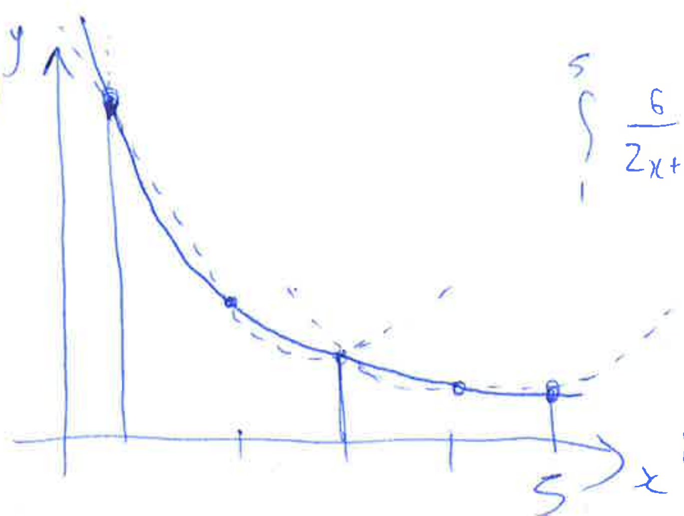
$\int_1^5 \frac{6}{2x+1}dx \approx \frac{h}{2}\left[f(1) + 2f(2) + 2f(3) + 2f(4) + f(5)\right]$

$= \frac{1}{2}\left[\frac{6}{3} + 2\cdot\frac{6}{5} + 2\cdot\frac{6}{7} + 2\cdot\frac{6}{9} + \frac{6}{11}\right]$

$= \frac{6}{2}\left[\frac{1}{3} + \frac{2}{5} + \frac{2}{7} + \frac{2}{9} + \frac{1}{11}\right]$

$= 3.9965$

(c)



$\int_1^5 \frac{6}{2x+1}dx \approx \frac{h}{3}\left[f(1) + 4f(2) + 2f(3) + 4f(4) + f(5)\right]$

$= \frac{6}{3}\left[\frac{1}{3} + \frac{4}{5} + \frac{2}{7} + \frac{4}{9} + \frac{1}{11}\right]$

$= 3.9088$

Note: The true answer is $\int_1^5 \frac{6}{2x+1}dx = 3\ln(11) - 3\ln(3) = 3.89785.$

(2) The most direct way to improve each estimate would be to increase the number of subintervals, because each approximate

$$\text{area} \longrightarrow \int_1^5 f(x)\,dx \quad \text{as} \quad n \to \infty.$$

(This is the definition of the definite integral.)

A sneakier idea might be to use a more sophisticated numerical integration ~~area~~ method, where the "top" of each ~~area~~ subinterval area better approximates the curve $y = f(x)$. The 3 methods here approximate the curve as a constant function $\longrightarrow$ linear function $\longrightarrow$ quadratic function. We could imagine methods which use cubic $\to$ quartic $\to$ quintic functions would get progressively more accurate. These methods exist, but we don't cover them in this course.