Math 20 Answer key **Quiz 3**

Name: **Date: 2/5/2014**

Directions: Calculators are allowed. Show all your working! Use the back of the page if you run out of space.

1. *(5 marks)* Calculate

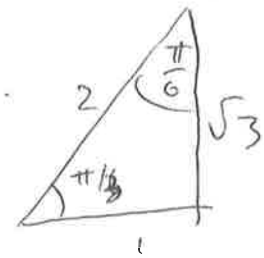
$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} e^{\frac{x}{\pi}} - \sin(x)dx$$

$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} e^{\frac{x}{\pi}} - \sin(x)\,dx$

$= \left[\pi e^{x/\pi} + \cos(x)\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$

$= \left(\pi e^{1/3} + \cos(\frac{\pi}{3})\right) - \left(\pi e^{1/4} + \cos(\frac{\pi}{4})\right)$

$= \pi(e^{1/3} - e^{1/4}) + \frac{1}{2} - \frac{\sqrt{3}}{2}$.



2. *(5 marks)* Using integration by substitution, find

$$\int_0^1 \frac{e^{2t}}{(3+e^{2t})^2}dt$$

$u = 3 + e^{2t}$

$\frac{du}{dt} = 2e^{2t}$

$du = 2e^{2t}dt$

$t = 0 \quad u = 4$.

$t = 1 \quad u = 3 + e^2$

$\int_0^1 \frac{e^{2t}}{(3+e^{2t})^2}dt = \frac{1}{2}\int_0^1 \frac{2e^{2t}dt}{(3+e^{2t})^2}$

$= \frac{1}{2}\int_4^{3+e^2} \frac{du}{u^2}$

$= \frac{1}{2}\frac{u^{-1}}{-1}\Big|_4^{3+e^2} = \frac{-1}{2}\left(\frac{1}{3+e^2} - \frac{1}{4}\right)$.

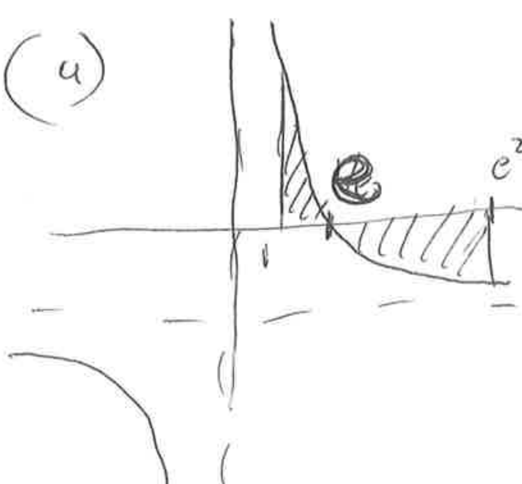
3. *(10 marks)* Find the area underneath the curve $f(x) = \frac{1}{x} - \frac{1}{e}$ between $x = 1$ and $x = e^2$. To do this, you will need to:

(a) Draw a graph of the function.

(b) Determine the value of x at which the curve crosses the x-axis (i.e., x when $f(x) = 0$).

(c) Calculate two definite integrals: the first from $x = 1$ to the value found in part (b), and the second from the value in part (b) to $x = e^2$.

(a)



(b) ($y = 0$ when $\frac{1}{x} - \frac{1}{e} = 0$, ie $\frac{1}{x} = \frac{1}{e}$, or $x = e$)

(c) $A = \int_1^e \frac{1}{x} - \frac{1}{e}dx - \int_e^{e^2} \frac{1}{x} - \frac{1}{e}dx$

$= \left[\ln(x) - \frac{x}{e}\right]_1^e - \left[\ln(x) - \frac{x}{e}\right]_e^{e^2}$

$= (1 - 1) - (0 - \frac{1}{e}) - (2 - e) + (1 - 1)$

$= \frac{1}{e} - 2 + e$