Directions: Calculators are allowed, but you shouldn't need to use your calculator. <u>Use your equals signs!</u>
Use the back of the page if you run out of space.

1. *(5 marks)* By using the definition of the tangent function and making the substitution $u = \cos(x)$, calculate

$$\int \tan(x)\,dx$$

Let $u = \cos(x)$

$\dfrac{du}{dx} = \sin(x)$

$du = -\sin(x)\,dx.$

So $\displaystyle\int \tan(x)\,dx$

$= \displaystyle\int \dfrac{\sin(x)}{\cos(x)}\,dx$

$= -\displaystyle\int \dfrac{-\sin(x)\,dx}{\cos(x)}$

$= -\displaystyle\int \dfrac{1}{u}\,du.$

$= -\ln|u| + c$

$= -\ln|\cos(x)| + c.$

2. *(5 marks)* Find the average of the function $f(x) = x\ln(x)$ on the interval from $x = 1$ to $x = e$. (You will need to use integration by parts here!)

Let $u = \ln(x)$ $\quad v = \dfrac{x^2}{2}$

$u' = \dfrac{1}{x}$ $\quad v' = x$

Then

$\bar{f} = \dfrac{1}{e-1}\displaystyle\int_1^e x\ln(x)\,dx$

$= \dfrac{1}{e-1}\left[[uv]_1^e - \displaystyle\int_1^e v\,u'\,dx\right]$

$= \dfrac{1}{e-1}\left[\left[\dfrac{x^2\ln(x)}{2}\right]_1^e - \dfrac{1}{2}\displaystyle\int_1^e \dfrac{x^2}{x}\,dx\right]$

$= \dfrac{1}{e-1}\left[\dfrac{e^2\ln(e)}{2} - \dfrac{\ln(1)}{2} - \dfrac{1}{2}\left[\dfrac{x^2}{2}\right]_1^e\right]$

$\dfrac{1}{e-1}\left\{\left[\dfrac{e^2}{2}\right] - \dfrac{1}{2}\left[\dfrac{e^2}{2}\right] + \dfrac{1}{2}\left[\dfrac{1^2}{2}\right]\right\}$

$= \left[\dfrac{e^2}{2} - \dfrac{e^2}{4} + \dfrac{1}{4}\right]_{e-1}^1$

$= \left[\dfrac{e^2 + 1}{4}\right]_{e-1}^1$

$= \dfrac{e^2 + 1}{4(e-1)}.$