Math 20 **Quiz 6**

Name: **Date: 3/12/2014**

Directions: Calculators are allowed, but you shouldn't need to use your calculator. Use your equals signs! Use the back of the page if you run out of space.

1. *(4 marks)* By replacing $-\infty$ with a and then taking an appropriate limit, calculate the improper integral

$$\int_{-\infty}^{-1} x^{-7/3}\,dx$$

$\int_{-\infty}^{-1} x^{-7/3}\,dx = \lim_{a\to-\infty} \int_a^{-1} x^{-7/3}\,dx$

$= \lim_{a\to-\infty} \left. \frac{x^{-4/3}}{-4/3} \right|_a^{-1}$

$= \lim_{a\to-\infty} \left(\frac{(-1)^{-4/3}}{-4/3} - \frac{a^{-4/3}}{-4/3} \right)$

$= \lim_{a\to-\infty} \left(-\frac{3}{4(-1)^{4/3}} - \frac{a^{-4/3}}{-4/3} \right)$

$= \lim_{a\to-\infty} \left(-\frac{3}{4} - \frac{a^{-4/3}}{-4/3} \right)$

[Tricky here: think about $((-1)^4)^{1/3} = (1^{1/3}) = 1$]

$= -\frac{3}{4}$.

2. *(4 marks)* Let

$$f(x,y) = \frac{\sqrt{6x+5y}}{\log_{10}(x)}.$$

Find:

(a) $f(10,8)$.

(b) Any values of x that are not in the domain of f (i.e., values of x for which the function is not defined).

(a) $f(10,8) = \frac{10}{\log_{10}(10)} = \frac{10}{1} = 10$.

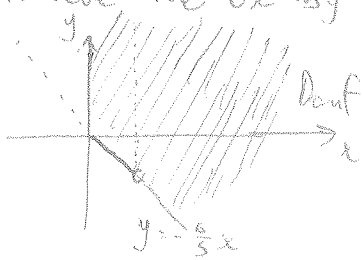
(b) Thinking about the $\log_{10}(x)$ in the denominator, this is undefined when $x \le 0$. (Like any log function is). This is all I wanted for your answer, but technically: we also need to have the $6x+5y$ under the $\sqrt{\ }$ be positive as well, so:

$6x+5y \ge 0$

$5y \ge -6x$

$y \ge -\frac{6}{5}x$.

so the domain is when both conditions are satisfied!



[And also note that $x \neq 1$, because otherwise we get 0 on the denominator of f!]

3. *(4 marks)* Sketch the plane $4x+2y+3z=24$. (Find the x-, y- and z-intercepts first!)

intercepts:

x: $4x = 24$, $x = 6$

y: $2y = 24$, $y = 12$

z: $3z = 24$, $z = 8$