

**Name:****Directions:** Calculators are allowed, but you shouldn't need to use your calculator. Use your equals signs!  
Use the back of the page if you run out of space.

1. (4 marks) Let

$$f(x,y) = y^2 e^{x+3y}.$$

Find the partial derivatives  $f_x$  and  $f_y$ .

$$f(x,y) = y^2 e^{(x+3y)}$$

$$f_x(x,y) = y^2 e^{(x+3y)}$$

Using the product rule,

$$\begin{aligned} f_y(x,y) &= 2ye^{x+3y} + y^2 e^{x+3y}(3) \\ &= ye^{(x+3y)}(2+3y). \end{aligned}$$

2. (6 marks) For the function  $f(x,y) = 9xy - x^3 - y^3 - 6$ , find(a) The values of  $x$  and  $y$  for which  $f_x = f_y = 0$ . (Fun fact: there are two solutions!)(b)  $f_{xx}$ ,  $f_{yy}$ , and  $f_{xy}$ .

$$(a) f(x,y) = 9xy - x^3 - y^3 - 6$$

$$f_x = 9y - 3x^2$$

$$f_y = 9x - 3y^2$$

$$\text{Let } f_x = 9y - 3x^2 = 0 \quad \dots (1)$$

$$\text{and } f_y = 9x - 3y^2 = 0 \quad \dots (2).$$

$$\text{From (1), } 9y = 3x^2 \\ y = \frac{x^2}{3}.$$

$$\text{Sub into (2): } 9x - 3\left(\frac{x^2}{3}\right)^2 = 0.$$

$$9x - \frac{3x^4}{9} = 0.$$

$$81x - 3x^4 = 0$$

$$27x - x^4 = 0.$$

$$x(27 - x^3) = 0.$$

$$\text{So } x = 0 \quad \text{or} \quad x^3 = 27 \\ x = 3.$$

$$\therefore y = 0 \quad \text{or} \quad y = \frac{9}{3} = 3.$$

$$(x,y) = (0,0) \quad \text{or} \quad (3,3).$$

$$(b) f_x = 9y - 3x^2$$

$$f_{xx} = -6x$$

$$f_y = 9x - 3y^2$$

$$f_{yy} = -6y$$

$$f_{xy} = 9 \quad (= f_{yx}!).$$