A PROOF OF THE THREE COLOUR THEOREM

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October 2008

Thesis submitted for the degree of Honours in ?????

SCHOOL OF MATHEMATICAL SCIENCES

THE UNIVERSITY OF ADELAIDE AUSTRALIA

Declaration

Except where stated this thesis is, to the best of my knowledge, my own work and my supervisor has approved its submission.

Signed by student:

Date:

Signed by supervisor:

Date:

Acknowledgements

Abstract

Contents

Chapter 1

Introduction

This is the introduction. Here is a reference to somebodies paper [1].

Chapter 2

Bundle gerbes

This is the next chapter and I have labelled it so I can refer to it later. Next is a displayed equation

$$P_{(y_1,y_2)} \otimes P_{(y_2,y_3)} \to P_{(y_1,y_3)}.$$

2.1 A section

Here is a new section and a displayed equation with a label.

$$d(\phi^{-1}(P), Z) = \phi^*(d(P, Y)). \tag{2.1.1}$$

It must be time for a theorem.

Theorem 2.1.1 ([2])**.** *If P and Q are bundle gerbes over M then*

1. $d(P^*) = -d(P)$ *and*
2. $d(P \otimes Q) = d(P) + d(Q)$.

Chapter 3

Stable isomorphism of bundle gerbes

3.1 Introduction

This Chapter is really good.

3.2 Referencing equations

The equation we did before can be referenced as (2.1.1) and the theorem as Theorem 2.1.1.

This is mathematics so we must need a definition.

Definition 3.2.1. Two bundle gerbes (P, Y) and (Q, Z) are called *stably isomorphic* if there are trivial bundle gerbes T_1 and T_2 such that

$$P \otimes T_1 = Q \otimes T_2.$$

There are equivalent definitions of stable isomorphism provided by the following proposition.

Proposition 3.2.2. *For bundle gerbes (P, Y) and (Q, Z) the following are equivalent.*

1. *P and Q are stably isomorphic*
2. *$P \otimes Q^*$ is trivial*
3. *$d(P) = d(Q)$.*

Proof. Clearly stably isomorphic bundle gerbes have the same Dixmier-Douady class because trivial bundles have the zero Dixmier-Douady class and the Dixmier-Douady class is additive over tensor products. So (1) implies (3). If $d(P) = d(Q)$ then $d(P \otimes Q^*) = d(P) - d(Q) = 0$. Hence $P \otimes Q^*$ is trivial ([2]). So (3) implies (2). Finally if $P \otimes Q^*$ is trivial then $Q \otimes Q^*$ is also trivial as it has zero Dixmier-Douady class and then $P \otimes (Q^* \otimes Q) = Q \otimes (P \otimes Q^*)$ so P and Q are stably isomorphic. So (2) implies (1). $\square$

From part (3) of Proposition 3.2.2 we see that stable isomorphism is an equivalence relation. Here is a list

- a C^* bundle $P_{\alpha\beta}$ over each intersection $U_\alpha \cap U_\beta$
- a trivialisation $\theta_{\alpha\beta\gamma}$ of the contracted product $P_{\beta\gamma} \otimes P^*_{\alpha\gamma} \otimes P_{\alpha\beta}$ over $U_\alpha \cap U_\beta \cap U_\gamma$
- the trivialisation satisfies $\delta(\theta) = 1$ over $U_\alpha \cap U_\beta \cap U_\gamma \cap U_\delta$.

3.2.1 Other related objects

We can have subsections.

Chapter 4

A great commuting diagram

In this Chapter we look at a commuting diagram done with the AMS package `amscd`. If that is not good enough you need to use something like `xypic`.

To include pictures use something like

```
\includegraphics[scale=0.25]{picture.pdf}
```

The file you are including is `picture.pdf` and the scale is a number between 0 and 1 that resizes the pciture by multiplying by that amount.

4.1 Here it is.

To calculate the Deligne cohomology we form the double complex:

$$\begin{array}{ccccccccc} \vdots & & \vdots & & \vdots & & & & \vdots \\ \delta\uparrow & & \delta\uparrow & & \delta\uparrow & & & & \delta\uparrow \\ \underline{U(1)}(Y_{\mathcal{U}}^{[3]}) & \xrightarrow{d\log} & \Omega^1(\Omega^{[3]}) & \xrightarrow{d} & \Omega^2(Y_{\mathcal{U}}^{[3]}) & \xrightarrow{d} & \cdots & \xrightarrow{d} & \Omega^q(Y_{\mathcal{U}}^{[3]}) \\ \delta\uparrow & & \delta\uparrow & & \delta\uparrow & & & & \delta\uparrow \\ \underline{U(1)}(Y_{\mathcal{U}}^{[2]}) & \xrightarrow{d\log} & \Omega^1(Y_{\mathcal{U}}^{[2]}) & \xrightarrow{d} & \Omega^2(Y_{\mathcal{U}}^{[2]}) & \xrightarrow{d} & \cdots & \xrightarrow{d} & \Omega^q(Y_{\mathcal{U}}^{[2]}) \\ \delta\uparrow & & \delta\uparrow & & \delta\uparrow & & & & \delta\uparrow \\ \underline{U(1)}(Y_{\mathcal{U}}) & \xrightarrow{d\log} & \Omega^1(Y_{\mathcal{U}}) & \xrightarrow{d} & \Omega^2(Y_{\mathcal{U}}) & \xrightarrow{d} & \cdots & \xrightarrow{d} & \Omega^q(Y_{\mathcal{U}}) \end{array} \tag{4.1.1}$$

The real Deligne cohomology is the cohomology of the double complex (4.1.1) which is calculated by forming the 'diagonal' complex

$$\underline{U(1)}(Y_{\mathcal{U}}) \xrightarrow{D} \underline{U(1)}(Y_{\mathcal{U}}^{[2]}) \oplus \Omega^1(Y_{\mathcal{U}}) \xrightarrow{D} \underline{U(1)}(Y_{\mathcal{U}}^{[3]}) \oplus \Omega^2(Y_{\mathcal{U}}^{[2]}) \oplus \Omega^3(Y_{\mathcal{U}}) \xrightarrow{D} \cdots \tag{4.1.2}$$

where the maps D are defined recursively by (for $g \in \underline{U(1)}(Y_{\mathcal{U}})$)

$$\begin{aligned} D(g) &= (\delta(g), d\log g) = (\delta(g), g^{-1}dg) \\ D(g,\omega^1) &= (\delta(g), \delta(\omega^1) - g^{-1}dg, d\omega^1) \\ D(g,\omega^1,\omega^2) &= (\delta(g), \delta(\omega^1) + g^{-1}dg, \delta(\omega^2) - d\omega^1, d\omega^2) \\ &\vdots \end{aligned}$$

Bibliography

[1] N.J. Hitchin, Lectures on special Lagrangian submanifolds. Preprint, Oxford 1999.

[2] M. K. Murray, Bundle gerbes, J. London Math. Soc. (2) **54** (1996), no. 2, 403–416.