2013-2014 Vacation Scholarship Projects

School of Mathematical Sciences

Project Titles

Characterising chaos

This project will study the concept of chaos from a mathematical viewpoint. Qualities which characterise chaos (e.g., sensitive dependence on initial conditions, presence of periodic orbits of all periods, uncountably many aperiodic orbits, and dense orbits) will be studied. Methods for analytically proving that a system is chaotic (presence of a "horseshoe," intersection of stable and unstable manifolds, Melnikov's method) will be applied to different systems arising in applications. *Prerequisites: a second-year differential equations course.*

Chaotic transport

In fluid systems which have chaotic fluid particle trajectories, the issue of quantifying the resulting chaotic transport has applications in, say, designing optimal micromixers and quantifying heat flux in the ocean. This project will review methods for quantifying chaotic transport, including Lyapunov exponents, lobe areas and a time-dependent flux, and perform numerical computations to establish connections between these methods. *Prerequisites: a second-year differential equations course, and programming capability in Matlab.*

Unsteady flow barriers

In unsteady (time-varying) flow fields, there are usually important invisible flow barriers which move with time. Examples include the boundary of an area ("the forbidden zone") near the coast of Florida into which the Deepwater Horizon oilspill did not leach, and the edge of the Antarctic Circumpolar Vortex ("the ozone hole"). This project will focus on understanding and using recently developing methods for identifying such flow barriers, and (depending on student background) will choose from George Haller's methods of minimum curve length increase, Gary Froyland's transfer operator methods and/or Jean-Luc Thiffeault's topological complexity of curves method. *Prerequisites: a second-year differential equations and fluid dynamics course, plus either a strong theoretical mathematics or programming background.*

I am also open to topics in other areas, to be agreed upon after discussion with me.

Supervisor: Dr Sanjeeva Balasuriya

I am happy to discuss possible topics for a research project in areas such as differential geometry, topology and complex analysis.

Supervisor: Dr David Baraglia

I am happy to supervise projects in the areas of finite geometry and combinatorics. Interested students can email me to arrange a meeting and we can discuss possible project ideas.

Supervisor: Dr Susan Barwick

Stochastic Modelling using Structured Markov Chains

In this project we will investigate the mathematical properties of some very recent stochastic models. These models have been developed from the basic principles used in a field known as “Matrix-analytic Methods” (or MAM) where simple exponentially distributed lifetimes are replaced by lifetimes from more complex distributions. When done carefully, the analysis of the whole model becomes matrix-based, rather than scalar-based, hence the name. Of course, this brings all sorts of challenges (for example, the square root operation no longer makes any sense) and requires a much closer connection to the physical model itself. This, and an associated emphasis on computational algorithms, are the main features of this area of stochastic modelling.

Supervisor: Professor Nigel Bean

Wave propagation in random media

Even the most simple wave problems become non-trivial and produce interesting behaviours when randomness is introduced. Students can investigate the effects of different types of randomness and in different settings, using numerical and/or analytical techniques.

Modelling sea ice dynamics & thermodynamics

Climate change is weakening and fragmenting the sea ice that covers vast areas of the Arctic Ocean. An important consequence is that the ice cover is now far more dynamic, and has more potential to melt in the summer and grow in the winter. Students can undertake a project to investigate this phenomenon.

Mathematical and experimental modelling

Combining mathematical and experimental models is a powerful way to accurately model real world phenomena. Students can conduct a project to develop a mathematical model of water waves and conduct simple wave tank experiments to validate the model.

Supervisor: Dr Luke Bennetts

Modelling Hirshsprung's disease:

Hirschsprung's disease is relatively common, affecting roughly 1 in 5000 newly born babies each year in Australia. The disease occurs when there is an incomplete formation of the nervous system in the gut. This project will explore both discrete and continuous mathematical models, which can help in determining the underlying mechanisms that cause the disease.

Supervisor: Dr Ben Binder

Here are some ideas for projects that might be of interest, with a rough indication of what background is likely to be needed.

Project 1: *How should one reverse-park a car? (Maths I)*

Project 2*:* *What is the optimal way to set traffic lights in a city? (Maths I)*

Project *3: All things considered, what is the optimal direction in which to point a solar panel? (Maths I + MV&CC)*

Project 4: *How much longer will every day be when the polar ice caps have melted? (Maths I + MV&CC)*

Project 5*: What shape will a perfect viscosity-free fluid take if it is in equilibrium, not rotating, and acted upon only by its own self-gravitation? What if it is assumed to be rotating (MV&CC)*

Project *6: The Dirichlet problem (Integration & Analysis, Topology & Analysis)*

Project 7*: The Riemann-Roch theorem (Complex Analysis)*

Project *8: Constant curvature rescaling of metrics on compact surfaces. (Integration & Analysis, Geometry of surfaces)*

I'm also open to suggestions from students with their own ideas, but we would have to discuss these in detail before I'd agree to supervise such a project.

Supervisor: Dr Nicholas Buchdahl

Nanoscaled oscillating systems

Nonoscaled structures such as carbon nanotubes and fullerenes undergo interactions described by van der Waals forces. At very small scales these interactions can lead to extreme accelerations, velocities and, in the case of oscillating systems, frequencies. By modelling the structures as surfaces with uniform atomic densities and the van der Waals interactions using a 6-12 Lennard-Jones potential, we can make predictions regarding these systems including deriving a formula for the frequency which is in good agreement with molecular dynamics simulations. In this project the student will look at models to calculate the force and predict the behaviour of various oscillating systems.

Geometries and geometric issues of nanostructures

It is clear from the various structures seen at the nanoscale that the complex interactions of these structures often lead to symmetric conformations. So in satisfying a minimum energy constraint the system often adopts a symmetric structure that shares the energetic costs of bending and stretching covalent bonds equally to all components in the structure. By assuming a symmetric conformation up front, it is possible to reduce fundamentally complex problems of molecular structure to problems with are more mathematically tractable and thereby derive results which can be confirmed by experiment and simulation and can also be used to predict ideal systems and novel structures in certain extreme cases. In this project the student will study models for nanostructures such as nanotubes, cones and spheres (buckyballs) with the aim to provide more precise predictions of structural parameters like length and radius.

Supervisor: Dr Barry Cox

Interactions in and between swarms and other biological aggregations

The collective behaviour of animals in swarms, or cells in tissues, is governed by the interactions between individuals in the group. We can use mathematical models to understand how different types of inter-individual interaction lead to different arrangements of cells in tissues, or movements of swarms. The models involve systems of two or more partial or integro-partial differential equations, and can be investigated using a combination of analytical and numerical methods.

Models for anisotropic materials

Many materials, particularly biological materials such as collagen gel, have a fibrous microstructure which affect how they respond to applied forces. This project will look at ways of accounting for this microstructure in mathematical models for the mechanics of these materials. A good knowledge of fluid mechanics will be an advantage.

Supervisor: Dr Edward Green

Mathematical physics

I am happy to supervise a summer research project in the areas of mathematical physics, differential geometry and representation theory. Possible topics include:

Project 1: Geometry of Black Holes and the Global Positioning System

Project 2: The Symmetric Group and Schur-Weyl Duality

Project 3: Möbius Transformations and Modular Forms

To discuss these or other projects, please email me to arrange a meeting.

Supervisor: Dr Pedram Hekmati

Special relativity beyond the speed of light

A recent theory describes the modified Lorentz transformations when two inertial frames are moving with a relative velocity which is in excess of the speed of light. The full implications of these new transformations have yet to be realized, and this project will examine some of the older theories for tachyonic motion, with reference to some particularly simple relativistic problems such as those which are usually examined for Einstein's theory of special relativity.

Modelling methane storage using nano-bottles

The traditional storage of methane involves the gas stored in a high-pressure environment. This presents environmental hazards for applications requiring a lot of methane (such as vehicle fuel, and domestic cooking and heating) because of the dangers of explosion. A new storage mechanism for methane involves nano-bottles, which combines the advantages of a high - pressure vessel and adsorbents but requires a lower pressure and thus present less risk of an accident. In this project, student would study models for the new storage possibilities for methane using nanotechnology.

Mathematical modelling on nanomechanics

Famous physicist Richard P. Feynman predicted in his historical talk 'There's plenty of room at the bottom' that the possibility of miniaturization and nano-devices assembling one atom at a time! And with the recent boom and success in the area of nanotechnology, such prediction seems within reach. Nanotechnology has shown to be useful in developing future drug delivery and high performance lithium battery, enhancing structural strength, advancing NEMS, etc. A deeper understanding of mechanics at the nanoscale is key to better design of nano-devices. In this project the student will concentrate on classical applied mathematical techniques for problems that would incur high cost to investigate experimentally or using computational methodologies.

Supervisor: Professor Jim Hill

Cluster analysis

The aim is to consider and compare a number of clustering methods, including k-means clustering, `trimmed' k-means clustering and polynomial histograms. The latter is a new method which generalises histograms for data with many variables and work well for determining the number of clusters. We apply these methods to different data, including data from flow cytometry which have up to 30 variables and more than 10,000 observations.

Density estimation for multivariate data

Density estimation is well understood for one- and two-dimensional data. We consider a number of methods for general multivariate data which estimate the data density by starting from a known density and updating this density one coordinate at a time. The methods include `marginal replacement', and non-Gaussian search methods. We compare the methodologies of the different techniques and apply them to the estimation of the density of multivariate data.

Supervisor: Associate Professor Inge Koch

The Classical Groups

The topic of the project are the classical matrix groups defined as invariance groups of bilinear and Hermitian forms over real, complex and quaternionic vector spaces. The aim is to study them as subminifolds in the vector space of n-by-n matrices and to establish some of the isomorphisms and covering maps for small dimensions n. Further exploration can be related to topological properties of the classical groups or to the quotient spaces arising from them.

Quaternions and Octonions

In a similar way how the complex numbers are constructed from the reals, the quaternions are constructed from the complex numbers and the octonions from the quaternions. Thus, both can be considered as generalisations of complex numbers to higher dimensions. Many interesting algebraic and geometric phenomena are related to the quaternions and octonions and some interesting groups are related to them. To explore these features and relations is the aim of the project.

Apart from these two topics I am happy to discuss and possible topic that is related to differential geometry.

Supervisor: Dr Thomas Leistner

Differential geometry

Mathematical physics

I am happy to discuss possible topics in the areas of differential geometry and mathematical physics. Interested students should email me and arrange a meeting.

Supervisor: Professor Michael Murray

Emergent distributions of noisy dynamics

In many applications the dynamics of important slow variables are entangled with fast uninteresting variables. This is especially true in noisy systems. This project is to extend analysis of the probability distributions to characterise emergent slow dynamics, and then to implement the generic analysis in a web service for all to use.

Dynamical systems view of adaptable moving meshes.

The idea is to solve PDE problems in time by moving the numerical grid in time to adaptively fit interesting structures, and avoid overly representing boring structures. This has been done for a long time, but the new view would be to see if dynamical system approach helps control stability and accuracy.

Supervisor: Prof Tony Roberts

Forward and backward branching processes for approximating epidemic dynamics

Branching processes are classical stochastic processes with wide applications. They provide fundamental insight to the evolution of an epidemic, over the entire course of an epidemic. This project will involve learning about the theory of branching processes, studying how they can be used for modeling epidemics, and implementing this theory and comparing results to simulations.

Transport risk pathways for emerging invasive species

Invasive species are a significant contributor to global biological change. We will develop mathematical models for forecasting the future risk of invasions through transport and migration pathways into Australia.

Supervisor: Dr Joshua Ross

Statistics and Biostatistics

I am happy to discuss potential projects in any areas of statistics and biostatistics. Please email me if you are interested and we can arrange a meeting to discuss topics.

Supervisor: Professor Patty Solomon

Diffusive transport of chemical signals

An unfertilized mammalian egg is surrounded by cumulus cells. On fertilization of the egg the cumulus cells move away from the egg and this is seen in high speed video as a travelling wave. It is thought that the cells respond to one or more chemical signals from locations on the surface of the egg. This project will explore travelling wave solutions of reaction-diffusion equations with the aim of developing a model to explain the behaviour of the surrounding cells after fertilization of an egg.

Supervisor: Dr Yvonne Stokes
