Institute for Geometry and its Applications
School of Mathematical Sciences
University of Adelaide

Workshop on Complex Geometry, 16–20 February 2009

ABSTRACTS OF TALKS

Emma Carberry, University of Sydney

Almost-complex tori in the 6-sphere

Octonionic multiplication defines a natural almost-complex structure on S^6 and almost-complex curves $M^2 \to S^6$ are rather pleasant examples of minimal surfaces. In particular, the cone over such a curve is associative and hence absolutely volume minimising. These almost-complex curves come in two types: they are either *isotropic* (in which case Bryant has shown they can be algebraically constructed from holomorphic maps) or they are *superconformal*. I shall describe a spectral curve approach to superconformal almost-complex tori; the main point of which is to also obtain an algebraic characterisation of these surfaces and hence study their moduli. An interesting feature is that the relevant abelian variety in this case is the intersection of two Prymians.

Jean-Pierre Demailly, Université de Grenoble (keynote speaker)

1. Plurisubharmonic singularities and approximation theorems

We will introduce basic invariants for plurisubharmonic singularities (Lelong numbers, singularity exponents) and will discuss a fundamental approximation result relying on the Ohsawa-Takegoshi extension theorem.

2. Analytic Zariski decomposition and related results in algebraic geometry

The goal of this second talk will be to outline an extension of Zariski decomposition, as known for surfaces, to higher dimensions, where it holds only approximately. This can be viewed as part of a new intersection theory framework which has deep applications to algebraic geometry, e.g. a characterization of the pseudoeffective cone and a characterization of projective algebraic varieties possessing a pseudoeffective canonical class.

3. Semicontinuity of singularities, estimates for Monge-Ampère operators, and existence of Kähler-Einstein metrics

This third part will discuss extensions of Nadel's criterion for the existence of Kähler-Einstein metrics on Fano orbifolds, and new results concerning log canonical thresholds in relation with estimates for Monge-Ampère operators. We will outline some important applications to local algebra as well as to birational geometry.

Michael Eastwood, Australian National University

The Penrose transform for complex projective space

Complex projective 2-space with its Fubini-Study metric is a self-dual Riemannian 4-manifold. As such it has a twistor space, a complex manifold that encodes its conformal structure. Twistor theory provides several automatic consequences via the Penrose transform (worked out by Buchdahl in this case). I shall review the twistor construction and its consequences for complex projective 2-space and present a generalisation and its consequences for complex projective n-space. These investigations are driven by problems in real integral geometry, which I shall also briefly discuss.

Peter Ebenfelt, University of California, San Diego (keynote speaker)

Rigidity and super-rigidity for CR mappings into hyperquadrics

The standard hyperquadrics in $\mathbf{C}^N$ are the CR analogs of flat Euclidean space in Riemannian geometry. There are many analogies, but also many differences, between isometric immersions into Euclidian space and CR mappings into hyperquadrics (that preserve, or at least do not add too much to, the signature of the Levi form). The main focus of these talks is to explore some aspects of these analogies and differences. One particular aspect is a rigidity phenomenon for CR mappings into hyperquadrics, limiting the number of mappings modulo the action of the automorphism group of the target hyperquadric. We will present some recent results in this directions, as well as some conjectures that remain open. We will also explain some of the main ideas that go into the proofs of these results. The main ingredients here include Cartan-Chern(-Moser) theory and some "sums/differences of squares" results.

The first lecture will be introductory, giving background and explaining some of the main results. At least a part of the second lecture will be dedicated to a "crash course" in Cartan-Chern(-Moser) theory. The third lecture will explain the ideas of proofs, present further applications of these ideas, and propose some conjectures.

Dariush Ehsani, Humboldt-Universität

Integral representations on Henkin-Leiterer domains

We review integral representations on bounded smooth strictly pseudoconvex domains and their applications to solving the $\bar{\partial}$-problem. We then discuss the problems in adapting the techniques in the smooth case to non-smooth strictly psuedoconvex domains, the so-called Henkin-Leiterer domains. We will give results of joint work with I. Lieb concerning weighted L^p estimates for the Bergman projection. We also discuss recent work involving C^k estimates for the canonical solution to the $\bar{\partial}$-problem as well as estimates in the case of higher order forms on such non-smooth domains.

Vladimir Ejov, University of South Australia

Degenerate hypersurfaces with a two-parametric family of automorphisms

This is a joint work with Gerd Schmalz (UNE) and Martin Kolar (Brno). We give a complete classification of Levi-degenerate hypersurfaces of finite type in $\mathbb{C}^2$ with two-dimensional symmetry groups. Our analysis is based on the classification of two-dimensional Lie algebras and an explicit description of isotropy groups for such hypersurfaces, which follows from the construction of Chern-Moser type normal forms at points of finite type.

John Erik Fornaess, University of Michigan

Finite type pseudoconvex domains

I will be discussing finite type pseudoconvex domains in $\mathbb{C}^3$. The problem is to understand the structure of the collection of complex curves which are tangent to high order. Hopefully a better understanding will make it a little easier to progress on developing the function theory of such domains, $\bar{\partial}$, peak functions, boundary behaviour of metrics, etc.

Franc Forstnerič, University of Ljubljana (keynote speaker)

A survey of the Oka principle and some recent applications

I will present the current status of the Oka principle and indicate some recent applications and open problems. In particular, I intend to discuss the parametric Oka principle for liftings, the ascent/descent of the Oka property in subelliptic Serre fibrations, the Oka principle over 1-convex spaces due to J. Prezelj, and the recent solution of the holomorphic Vaserstein problem due to B. Ivarsson and F. Kutzschebauch.

Rod Gover, University of Auckland

The Fefferman space over a CR manifold and prolonged differential systems

To a codimension 1 non-degenerate CR manifold one can canonically associate a conformal manifold known as the Fefferman space. The idea of this construction is to relate the local and global CR geometric objects to their conformal counterparts. For example one may construct CR invariant powers of the sub-Laplacian and CR Q-curvature this way. If one calculates directly with tensors then relating objects on the two spaces rapidly becomes complicated. Appropriate prolonged systems of a tractor calculus yield a vastly simpler picture. The tractor connection is straightforward to construct, and this then yields the normal Cartan bundle and so all associated structures. We obtain a conformal characterisation of the Fefferman space as simply a certain holonomy reduction of the conformal tractor bundle. It is then straightforward to lift natural CR tensors and equations and descend appropriate objects. For example we explain the extent to which a Fefferman space may admit an Einstein metric and so conclude a notion of Einstein for CR manifolds (recovering a result of Leitner). This is joint work with Robin Graham and Andreas Cap.

Adam Harris, University of New England

Aspects of the Kodaira-Spencer equation for complex structures

Kodaira-Spencer deformations of the complex structure of a given manifold are themselves almost complex. The integrability condition that must further be satisfied by these new structures in order to qualify as fully complex takes the form of a partial differential equation on the space of deformation tensors, which is quite familiar in appearance to students of Cartan geometry. Conditions under which this equation is solvable provide information about the local structure of the parameter space of complex deformations of the original complex manifold. In this talk I will briefly survey some famous results in this field, due to Kodaira-Spencer, Kuranishi, and Bogomolov-Tian-Todorov, before presenting an approach to the equation as it relates to the smooth locus of a specific class of Kähler varieties with isolated singularity.

Alexander Isaev, Australian National University

Classical symmetries of complex manifolds

We consider complex manifolds that admit classical symmetries, that is, manifolds endowed with almost effective actions by holomorphic transformations of classical simple real Lie groups. Here we say that a real Lie group is a classical simple group if it is connected and its Lie algebra is a real form of a classical simple complex Lie algebra. Examples of such manifolds are given by open subsets of complex flag manifolds G/P invariant under the action of a real form of G. In my talk I will present a complete explicit classification of manifolds with classical symmetries in some natural situations. The work is joint with Alan Huckleberry.

Kang-Tae Kim, Pohang University of Science and Technology

CR hypersurfaces with a CR contraction

Let (M,p) be a germ of a smooth CR manifold with CR codimension 1 (i.e., hypersurface type). The main result I would like to discuss in the talk is as follows:

Theorem [Kim-Yoccoz] If (M,p) admits a CR contraction (i.e., a CR self-map of M contracting at p), then:
(1) The CR germ (M,p) is embedded (as a CR hypersurface).
(2) The embedding can be chosen so that (M,p) is defined by a weighted homogeneous polynomial.
(3) The monomial degrees of the defining polynomial is determined by the resonance set of the contraction.

This result gives therefore a local characterization of smooth CR manifolds with a CR contraction. The preprint is available at `http://www.arxiv.org`, arXiv:0807.0482v1 [math.CV].

Ilya Kossovskiy, Australian National University

Homogeneous hypersurfaces in $\mathbb{C}^3$ associated with the automorphism group of the 4-dimensional CR-cubic

We classify local transitive actions on $\mathbb{C}^3$ of a solvable Lie algebra, known as the infinitesimal automorphism algebra of the 4-dimensional CR-cubic in $\mathbb{C}^3$, up to local biholomorphic equivalence. It turns out that there are four pairwise non-equivalent actions of this algebra. A full classification of the orbits of these actions is presented. Among these four collections of orbits one can find both well-known hypersurfaces in $\mathbb{C}^3$ (hyperquadric, hyperplane, the future light cone) and essentially new examples of homogeneous hypersurfaces.

Finnur Lárusson, University of Adelaide

Siciak-Zahariuta extremal functions, analytic discs and polynomial hulls

I will report on joint work with Ragnar Sigurdsson. We prove two disc formulas for the Siciak-Zahariuta extremal function of an arbitrary open subset of $\mathbb{C}^n$. We use these formulas to characterise the polynomial hull of an arbitrary compact subset of $\mathbb{C}^n$ in terms of analytic discs. Similar results in previous work of ours required the subsets to be connected. It turned out that a different method of proof was needed in the general case.

Jürgen Leiterer, Humboldt-Universität

On the compactification of concave ends

This is a talk about joint work with Martin Brumberg.

Let X be a 1-*corona*, i.e., a complex manifold together with a strictly plurisubharmonic $\mathcal{C}^\infty$-function ρ defined on X and with values in an open interval $]a,b[$, $a \in \mathbb{R} \cup \{-\infty\}$, $b \in \mathbb{R} \cup \{\infty\}$, such that the sets $\{\alpha \le \rho \le \beta\}$, $a < \alpha < \beta < b$, are compact. We say that the concave end of X can be *compactified* if X is (biholomorphic to) an open subset of a complex space $\widehat{X}$ such that, for $a < c < b$, the set $(\widehat{X} \setminus X) \cup \{a < \rho \le c\}$ is compact. For $\dim X \ge 3$ it was proved by H. Rossi that this is always possible.

Now let $\dim X = 2$. Then this is not true. There are different counterexamples. We prove that then the concave end of X can be compactified if and only if $H^1(X)$ is Hausdorff, i.e., if the space of exact $\mathcal{C}^\infty_{0,1}$-forms is closed with respect to uniform convergence on compact sets together with all derivatives. (That this condition is necessary is well known by the Andreotti-Vesentini separation theorem.) More precisely, we prove that then X can be embedded into $\mathbb{C}^3$ and the existence of a compactification follows from the Harvey-Lawson theorem.

Amnon Neeman, Australian National University (keynote speaker)

Grothendieck duality, the modern way

We will describe recent progress on the subject of Grothendieck duality. We will begin with Serre's duality theorem, for a vector bundle on a compact, complex manifold. We will recall Grothendieck's relative version, for proper holomorphic maps $X \to Y$. We will also describe the version in algebraic geometry, valid for sufficiently nice schemes.

Then we will quickly describe some recent work which has shed a completely new light on the old results; at the moment the recent work is confined to the setting of algebraic geometry. It is not clear what the holomorphic analog should be, let alone how to prove it. I will try to give an overview of what we know (for schemes).

Paul Norbury, University of Melbourne

Magnetic monopoles on manifolds with boundary

Kapustin and Witten show that geometric Langlands—an equivalence involving holomorphic objects over Riemann surfaces—naturally arises in gauge theory. They associate a Hecke modification of a holomorphic bundle over a Riemann surface to a singular monopole on a Riemannian surface times an interval satisfying prescribed boundary conditions. I will describe this, and prove existence and uniqueness of such monopoles for given Hecke modification data confirming the underlying geometric invariant theory principle.

Gerd Schmalz, University of New England

Holomorphicity of functions annihilated by one singular vector field

I will talk about the following generalization of Forelli's theorem. Suppose X is a holomorphic vector field with singular point at p, such that X is linearizable at p and the corresponding matrix A is diagonalizable. Assume that the eigenvalues of A have pairwise positive ratios. Then any function ϕ that has an asymptotic Taylor expansion at p and is holomorphic along the complex integral curves of X is holomorphic in a neighborhood of p. The assumption for the ratios of the eigenvalues to be positive reals is necessary. The original theorem by Forelli covers the case when A is the unit matrix. This is joint work with Kang-Tae Kim and Evgeny Poletsky.