

IGA/AMSI Workshop on

The Mathematical Implications of Gauge-String Dualities

The University of Adelaide, March 5–9, 2012

Lectures of Rajesh Gopakumar:

- *What are Gauge-String Dualities?*

In this introduction, I will give a broad overview of the topic of Gauge-String duality (a.k.a. AdS/CFT correspondence) which is one of the most engaging topics of current research in theoretical physics. I will try to convey some of the physical motivation, context and importance as well as the mathematical significance of the subject. In particular, the famous Kontsevich derivation of the intersection numbers on the moduli space of Riemann surfaces, turns out to be a special case of such a duality.

- *From Knots to Counting Curves*

To further illustrate the power of these ideas I will describe an unexpected connection between the theory of knots and three manifold invariants (as reformulated by Witten in terms of Chern-Simons quantum field theory) and enumerative invariants of (a class of) Calabi–Yau 3-folds.

- *From Gauge Theories to String Theories*

I proceed to describe a general picture of how gauge theories (in the so-called large N limit) can reorganize themselves into string theories. Underlying this is a cell decomposition of the moduli space of punctured Riemann surfaces and its Strebel parametrisation.

- *What is the Simplest Gauge-String Duality?*

I will argue that one of the most explicit illustrations of how the above reorganization is realized is in a conjectured relation between a Gaussian matrix integral and topological strings on $\mathbb{C}P^1$. Belyi maps and arithmetic Riemann surfaces seem to play a very special role in this case.

- *Dualities for 2d CFTs*

In the final lecture I describe a new direction in which special solvable $2d$ conformal field theories (with so-called extended W_N symmetry) are dual to theories involving gravity together with a whole tower of higher spin gauge fields in three dimensional hyperbolic space (AdS_3).

Other invited speakers:

1. **Peter Bouwknegt**, Australian National University

Title: AKSZ construction of topological open p -brane action and Nambu brackets

Abstract: We review the AKSZ construction as applied to the topological open membranes and Poisson sigma models. We describe a generalization to open topological p -branes and Nambu-Poisson sigma models. This talk is based on joint work with Brano Jurco.

2. **Omar Foda**, University of Melbourne

Title: Slavnov's determinant and YM 3-point functions

Abstract: I wish to recall the definition of Slavnov's determinant expression for the scalar product of a Bethe eigenstate and a generic state in Heisenberg spin-1/2 chains, which is a distinguished object in exact solutions in statistical mechanics.

Next, I wish to explain how, due to the integrability of weakly-coupled conformally-invariant pure-gauge Yang-Mills theories, Slavnov's determinant recently showed up in computations of certain 3-point functions of local composite operators.

Work in collaboration with C Ahn, R Nepomechie and M Wheeler.

3. **Sergei Kuzenko**, University of Western Australia

Title: Hyperkähler geometry of extended supersymmetric sigma models in AdS_4

Abstract: It is a well-known result of supersymmetric field theory that hyperkähler manifolds are in one-to-one correspondence with four-dimensional $N = 2$ supersymmetric sigma models in Minkowski space. Given such a sigma model, the associated hyperkähler manifold is its target space. Minkowski space-time is a maximally supersymmetric solution of $N = 2$ supergravity without a cosmological term in four dimensions. In the presence of a cosmological term, the maximally supersymmetric solution of $N = 2$ supergravity is the so-called anti-de Sitter space (AdS_4). In this talk we show that the target space geometry of $N = 2$ supersymmetric sigma models in AdS_4 differs from that in Minkowski target space in AdS_4 proves to be a non-compact hyperkähler manifold restricted to possess a special Killing vector field which generates an $SO(2)$ group of rotations on the two-sphere of complex structures and necessarily leaves one of them invariant. Such hyperkähler metrics can be generated using $N = 2$ superspace techniques and a certain duality transformation.

4. **Siye Wu**, University of Hong Kong

Title: Index bundle gerbes and moduli spaces

Abstract: We construct the index bundle gerbe associated to a family of self-adjoint Dirac-type operators, refining a construction of Segal. In a special case, we also give

a geometric description which agrees with the above analytic construction. Finally, we apply the result to certain moduli spaces associated to Riemann surfaces. This is a joint work with P. Bouwknegt and V. Mathai.