Dualities in Field Theories and the Role of K-Theory

Jonathan Rosenberg

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Abstracts of the Lectures

Lecture 1 The basic idea of dualities in field theories and T-duality

It is now known (or in some cases just believed) that many quantum field theories exhibit *dualities*, equivalences with the same or a different theory in which things appear very different, but the overall physical implications are the same. We will discuss some of these dualities from the point of view of a mathematician, and then give a (simplified) outline of the case of string theory and T-duality ("target space" duality). This duality is closely related to other dualities in string theory, such as mirror symmetry.

Lecture 2 K-Theory and its relevance to physics

K-theory is the extraordinary cohomology theory that arises out of classifying vector bundles. It thus has a very simple geometric interpretation. We will discuss some of the ways vector bundles arise in physics and how the study of "charge conservation" forces one to consider the role played by K-theory.

Lecture 3 Basics of C^*-algebras and crossed products

This lecture will be an elementary introduction to some aspects of C^*-algebras, which are needed for formulating the idea of a "noncommutative space." I will also explain what crossed products are, give some examples, and illustrate what they are good for.

Lecture 4 Continuous-trace algebras and twisted K-theory

We will review the definition of *continuous-trace* algebras, C^*-algebras locally Morita equivalent to commutative algebras, and the basic classification theory for them as developed by Dixmier and Douady. The K-theory of continuous-trace algebras is known as twisted K-theory. We will explain why, in the presence of background fluxes, the K-groups appearing in string theory must be replaced by twisted K-groups.

Lecture 5 The topology of T-duality and the Bunke-Schick construction

This lecture will introduce the idea, originally due to Bouwknegt, Evslin, and Mathai, of *topological T-duality.* While T-duality for physicists is a *metric notion*, this concentrates on just the *topological* aspect of T-duality, the part that is independent of the metric. This can be viewed as the "leading term" in the physicists' (metric) T-duality. Bunke and Schick gave a very nice way, which we will also discuss, of *axiomatizing* topological T-duality.

Lecture 6 T-duality via crossed products

An alternative approach to topological T-duality, due to Mathai and the author, depends on crossed products and continuous-trace algebras, which were introduced in Lectures 3 and 4. We will quickly explain the basic idea and how it sometimes leads to *non-classical* T-duals in the higher-dimensional case.

Lecture 7 Problems presented by S-duality

S-duality is in many ways a much more mysterious duality than T-duality, but it seems to involve very deep mathematics. (For example, Kapustin and Witten have related it to the Langlands program.) In this lecture we will discuss some of the puzzles it raises in terms of K-theory constraints, and a possible approach to handling some of them, due to Mendez-Diez and the author.

Lecture 8 The AdS/CFT correspondence and problems it raises

Still another mysterious duality in string theory is the AdS/CFT correspondence, which was the main topic of the parallel lecture series by Gopakumar. The main question we will try to address is how to relate the K-theoretic classification of branes in string theory to invariants of a similar nature in gauge theory.

Lecture 9 KR-theory and some KR calculations

KR-theory, introduced by Atiyah, the K-theory of complex vector bundles with a conjugate-linear involution compatible with a fixed involution on the base. This is the appropriate K-theory for studying, for example, algebraic varieties defined over $\mathbb{R}$. We will discuss a few facts about this variant of K-theory and discuss some calculations of it.

Lecture 10 T-duality for orientifolds and applications of KR-theory

In the final talk, we will discuss string theory on *orientifolds*, spacetime manifolds equipped with an involution. In such a theory, charges take their values in KR-theory. We will concentrate on the case where the spacetime is an elliptic curve defined over $\mathbb{R}$ (crossed with $\mathbb{R}^8$ with trivial involution) and discuss how T-duality matches up with the KR calculations. This is joint work with Doran and Mendez-Diez.

Other invited speakers:

1. **Alan Carey**, Australian National University

   Title: Spectral flow using Kasparov

   Abstract: This talk is about a conjectural approach to spectral flow in unbounded Kasparov modules.

2. **Nora Ganter**, University of Melbourne

   Title: $196,884 = 196,883 + 1$

   Abstract: I will give an introduction to Moonshine and, time permitting, highlight some more recent directions.

3. **Wend Werner**, University of Münster

   Title: K-theory and the classification of symmetric spaces

   Abstract: The aim of this talk is to indicate how the classification of symmetric spaces can be achieved by K-theoretical methods. We focus on Hermitian symmetric spaces of non-compact type. By its Harish–Chandra realization and Koecher theory, a space of this type allows for a representation as the open unit ball of a so called JB^*-triple system, a concept generalizing C^*-algebras, in which, roughly, the binary operation is replaced by a triple product. We define K-theory for the latter. K-groups have to be ordered and marked by certain classes (which, essentially, correspond to the root lattice of an underlying Lie algebra) in order to become classifying.

   Spaces classifiable by this K-theoretic invariant include inductive limits of finite dimensional JB^*-triples as well as spaces strongly connected to Kac–Moody algebras of a generalized type.

   (Joint with D. Bohle)

4. **Craig Westerland**, University of Melbourne

   Title: Some higher chromatic analogues of the J-homomorphism

   Abstract: Different cohomology theories "see" different parts of the stable homotopy groups of spheres. Singular cohomology, for instance, detects all maps from a sphere to itself using the notion of degree. K-theory detects a large swath of homotopy known as the "image of J," which can be described very geometrically using the relation between a vector bundle and its unit sphere-bundle. In this talk, which covers work that is very much in progress, I will discuss an analogue of the image of J for higher chromatic homotopy theory – the part seen by the Morava K-theories, $K(n)$. The result lacks the charming geometry of the J-homomor notion of "homotopy group," but will hopefully give us insight into the $K(n)$-local homotopy category.